What do we know about the Standard Model?

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Lecture 4

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The Standard Model Works

- Any discussion of the Standard Model has to start with its success
- This is unlikely to be an accident!

Unitarity

- Consider 2 $\rightarrow$ 2 elastic scattering

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}|A|^2$$

- Partial wave decomposition of amplitude

$$A = 16\pi \sum_{l=0}^{\infty} (2l+1) P_l(\cos\theta) a_l$$

- a_l are the spin *l* partial waves

Unitarity

- $P_l(\cos\theta)$ are Legendre polynomials:

$$\int_{-1}^{1} dx P_l(x) P_{l'}(x) = \frac{2\delta_{l,l'}}{2l+1}$$

$$\sigma = \frac{8\pi}{s} \sum_{l=0}^{\infty} (2l+1) \sum_{l'=0}^{\infty} (2l'+1) a_l a_{l'}^{*} \int_{-1}^{1} d\cos\theta P_l(\cos\theta) P_{l'}(\cos\theta)$$

$$= \frac{16\pi}{s} \sum_{l=0}^{\infty} (2l+1) \left| a_l \right|^2$$

Sum of positive definite terms

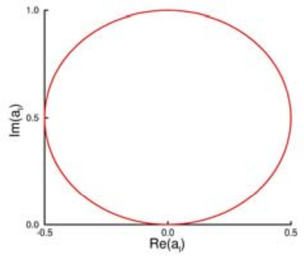
More on Unitarity

- Optical theorem $\sigma = \frac{1}{s}\mathrm{Im}[A(\theta=0)] = \frac{16\pi}{s}\sum_{l=0}^{\infty}(2l+1)|a_l|^2$

$$\boxed{\mathrm{Im}(a_l) = |a_l|^2}$$

Optical theorem derived assuming only conservation of probability

- Unitarity requirement:



$$|\mathrm{Re}(a_l)| \le \frac{1}{2}$$

More on Unitarity

- Idea: Use unitarity to limit parameters of theory

Cross sections which grow with energy always violate unitarity at some energy scale

Example 1: $W^+W^- \to W^+W^-$

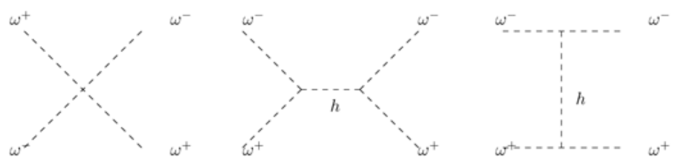
- Recall scalar potential (Include Goldstone Bosons in Unitarity gauge)

$$V = \frac{M_h^2}{2} h^2 + \frac{M_h^2}{2v} h\left(h^2 + z^2 + 2\omega^+\omega^-\right) + \frac{M_h^2}{8v^2}\left(h^2 + z^2 + 2\omega^+\omega^-\right)^2$$

- Consider Goldstone boson scattering:

$\omega^+\omega^- \to \omega^+\omega$

$$iA(\omega^+\omega \to \omega^+\omega^-) = -2i\frac{M_h^2}{v^2} + \left(-i\frac{M_h^2}{v}\right)^2 \frac{i}{t - M_h^2} + \left(-i\frac{M_h^2}{v}\right)^2 \frac{i}{s - M_h^2}$$



$\omega^{+}\omega^{-}\rightarrow\omega^{+}\omega^{-}$

- Two interesting limits:
  - s, t >> M_h^2

$$A(\omega^{+}\omega^{-}\rightarrow\omega^{+}\omega^{-})\rightarrow -2\frac{M_h^2}{v^2}$$

$$\boxed{a_0^0 \rightarrow -\frac{M_h^2}{8\pi v^2}}$$

  - s, t << M_h^2

$$A(\omega^{+}\omega^{-}\rightarrow\omega^{+}\omega^{-})\rightarrow -\frac{u}{v^2}$$

$$\boxed{a_0^0 \rightarrow -\frac{s}{32\pi v^2}}$$

Use Unitarity to Bound Higgs

$$\left|\mathrm{Re}(a_l)\right| \le \frac{1}{2}$$

- High energy limit:

$$\boxed{a_0^0 \to -\frac{M_h^2}{8\pi v^2}}$$

$\boxed{M_h < 800 \text{ GeV}}$

- Heavy Higgs limit

$$\boxed{a_0^0 \to -\frac{s}{32\pi v^2}}$$

$E_c \sim 1.7$ TeV

$\rightarrow$ New physics at the TeV scale

Can get more stringent bound from coupled channel analysis

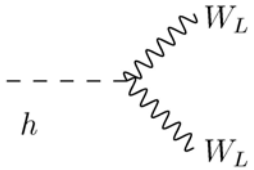
Electroweak Equivalence Theorem

$$A(V_L^1 \ldots V_L^N \rightarrow V_L^1 \ldots V_L^{N'}) = (i)^N (-i)^{N'} A(\omega_1 \ldots \omega_N \rightarrow \omega_1 \ldots \omega_{N'})$$
$$+O\left(\frac{M_W^2}{E^2}\right)$$

This is a statement about scattering amplitudes, NOT individual Feynman diagrams

Plausibility argument for Electroweak Equivalence Theorem

- Compute $\Gamma(h\rightarrow W_L^+ W_L^-)$ for $M_h >> M_W$



$$\varepsilon_L = \frac{1}{M_W}(|\vec{p}|,0,0,p_0) \approx \frac{p}{M_W}$$

$$\begin{aligned} iA = &\quad -igM_W g^{\mu\nu}\varepsilon_\mu \varepsilon_\nu \\ = &\quad -igM_W \frac{p_+ \cdot p_-}{M_W^2} \\ \rightarrow &\quad -ig\frac{M_h^2}{2M_W} \end{aligned}$$

$$\Gamma(h \rightarrow W_L^+ W_L^-) \approx \frac{G_F M_h^3}{8\pi\sqrt{2}}$$
$$= \Gamma(h \rightarrow \omega^+ \omega^-)$$

$\boxed{\Gamma(h\rightarrow WW) \approx M_h \text{ for } M_h \approx 1.4 \text{ TeV}}$

Landau Pole

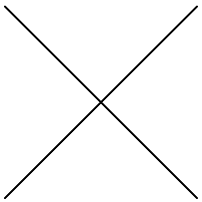
- M_h is a free parameter in the Standard Model
- Can we derive limits on the basis of consistency?
- Consider a scalar potential:

$$V = \frac{M_h^2}{2} h^2 + \frac{\lambda}{4} h^4$$

- This is potential at electroweak scale
- Parameters evolve with energy in a calculable way

Consider hh→hh

- Real scattering, $s+t+u=4M_h^2$
- Consider momentum space-like and off-shell: $s=t=u=Q^2<0$
- Tree level: $iA_0=-6i\lambda$



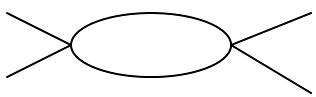
hh→hh, #2

- One loop:

$$iA_s = (-6i\lambda)^2 \frac{1}{2}\int \frac{d^n k}{(2\pi)^2} \frac{i}{k^2 - M_h^2} \frac{i}{(k+p+q)^2 - M_h^2}$$

$$= \frac{9\lambda^2}{8\pi^2}(4\pi\mu^2)\Gamma(\varepsilon)\left(M_h^2 - Q^2 x(1-x)\right)^{-\varepsilon}$$

- $A = A_0 + A_s + A_t + A_u$



$$A = -6\lambda\left(1 + \frac{9\lambda}{16\pi^2}(4\pi\mu^2)\Gamma(\varepsilon)\left(M_h^2 - Q^2 x(1-x)\right)^{-\varepsilon} + \ldots\right)$$

hh→hh, #3

- Sum the geometric series to define running coupling

$$A = -6\lambda\left(1 + \frac{9\lambda}{16\pi^2}\log\frac{Q^2}{M_h^2}\right) + \ldots$$

$$A = \frac{6\lambda}{1 - \frac{9\lambda}{8\pi^2}\log\left(\frac{Q}{M_h^2}\right)} \equiv 6\lambda(Q)$$

- $\lambda(Q)$ blows up as $Q\to\infty$ (called Landau pole)

hh→hh, #4

- This is independent of starting point
- BUT…. Without $\lambda\phi^4$ interactions, theory is non-interacting
- Require quartic coupling be finite

$$\frac{1}{\lambda(Q)} > 0$$

hh→hh, #5

- Use $\lambda=M_h^2/(2v^2)$ and approximate $\log(Q/M_h) \to \log(Q/v)$
- Requirement for $1/\lambda(Q)>0$ gives upper limit on M_h

$$M_h^2 < \frac{32\pi^2 v^2}{9\log\left(\frac{Q^2}{v^2}\right)}$$

- Assume theory is valid to 10^{16} GeV
  - Gives upper limit on M_h< 180 GeV
- Can add fermions, gauge bosons, etc.

High Energy Behavior of λ

- Renormalization group scaling $\frac{1}{\lambda(Q)}=\frac{1}{\lambda(\mu)}+(\ldots)\log\left(\frac{Q}{\mu}\right)$

$$\boxed{16\pi^2\frac{d\lambda}{dt}=12\lambda^2+12\lambda g_t^2-12g_t^4+(gauge)}$$

$$t\equiv\log\left(\frac{Q^2}{\mu^2}\right) \qquad g_t=\frac{M_t}{v}$$

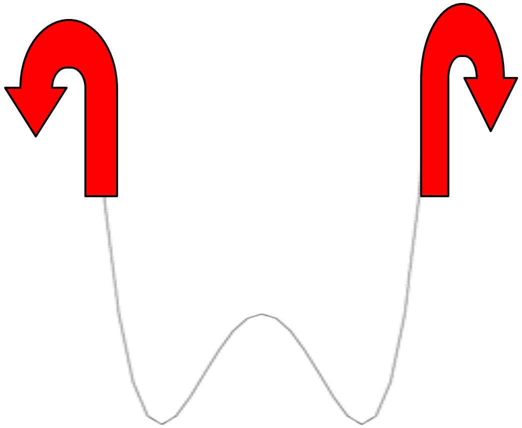
- *Large λ (Heavy Higgs):* self coupling causes λ to grow with scale
- *Small λ (Light Higgs):* coupling to top quark causes λ to become negative

Does Spontaneous Symmetry Breaking Happen?

- SM requires spontaneous symmetry
- This requires $V(v) < V(0)$
- For small λ

$$\boxed{16\pi^2 \frac{d\lambda}{dt} \approx -16 g_t^4}$$

- Solve $\boxed{\lambda(\Lambda) \approx \lambda(v) - \frac{3g_t^4}{4\pi^2} \log\left(\frac{\Lambda^2}{v^2}\right)}$



Does Spontaneous Symmetry Breaking Happen? (#2)

- $\lambda(\Lambda) > 0$ gives lower bound on M_h

$$\boxed{M_h^2 > \frac{3v^2}{2\pi^2} \log\left(\frac{\Lambda^2}{v^2}\right)}$$

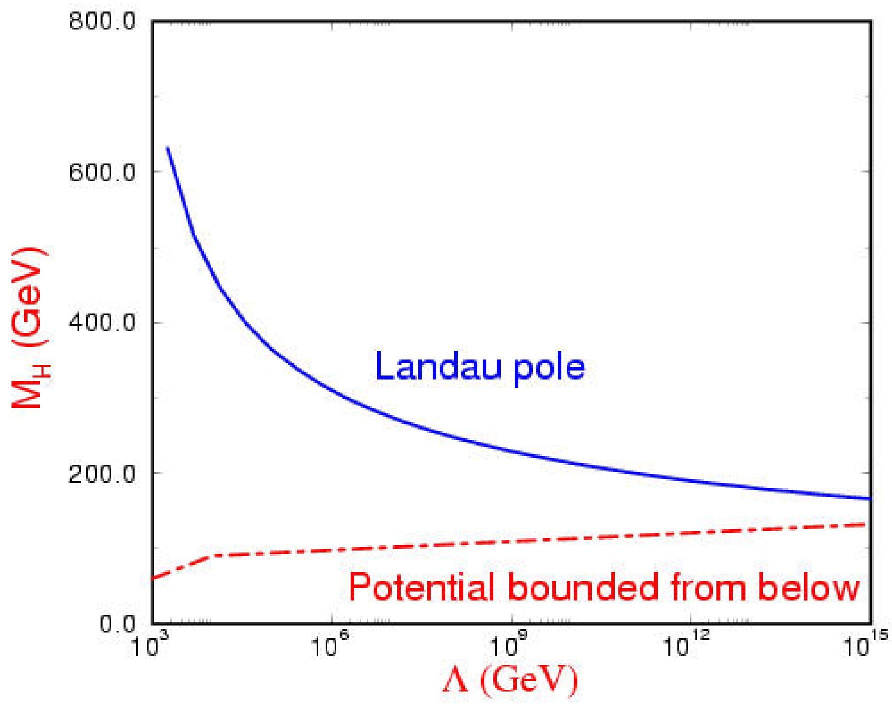
- If Standard Model valid to 10^{16} GeV

$$\boxed{M_h > 130\ GeV}$$

- For any given scale, Λ, there is a theoretically consistent range for M_h

Bounds on SM Higgs Boson

- If SM valid up to Planck scale, only a small range of allowed Higgs Masses



Problems with the Higgs Mechanism

- We often say that the SM cannot be the entire story because of the quadratic divergences of the Higgs Boson mass

Masses at one-loop

- First consider a fermion coupled to a massive complex Higgs scalar

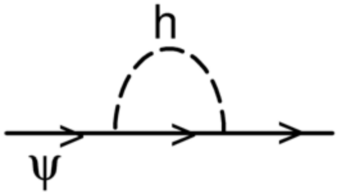
$$L = \overline{\Psi}(i\partial)\Psi + \left|\partial_\mu \phi\right|^2 - m_s \left|\phi\right|^2 - \left(\lambda_F \overline{\Psi}_L \Psi_R \phi + h.c.\right)$$

- Assume symmetry breaking as in SM:

$$\phi = \frac{(h+v)}{\sqrt{2}} \qquad\qquad m_F = \frac{\lambda_F v}{\sqrt{2}}$$

Masses at one-loop, #2

- Calculate mass renormalization for Ψ



$$-i\Sigma_F(p) = \left(\frac{-i\lambda_F}{\sqrt{2}}\right)^2 (i)^2 \int \frac{d^4k}{(2\pi)^4} \frac{k + m_F}{[k^2 - m_F^2][(k-p)^2 - m_s^2]}$$

Renormalized fermion mass

$$\delta m_F = \Sigma_F(p)\Big|_{p=m_F}$$

$$= i\frac{\lambda_F^2}{32\pi^4}\int_0^1 dx \int d^4k' \frac{m_F(1+x)}{[k'^2 - m_F^2 x^2 - m_s^2(1-x)]^2}$$

- Do integral in Euclidean space

$$k_0 \to ik_4$$

$$d^4k' \to i d^4 k_E$$

$$k'^2 = k_0^2 - \left|\vec{k}\right|^2 \to k_4^2 - \left|\vec{k}\right|^2 = -k_E^2$$

$$\int d^4 k_E\, f(k_E^2) = \pi^2 \int_0^{\Lambda^2} y\, dy\, f(y)$$

Renormalized fermion mass, #2

- Renormalization of fermion mass:

$$\delta m_F = -\frac{\lambda_F^2 m_F}{32\pi^2}\int_0^1 dx\ (1+x)\int_0^{\Lambda^2}\frac{y\,dy}{[y+m_F^2 x^2+m_s^2(1-x)]^2}$$

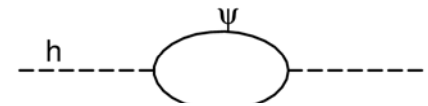
$$= -\frac{3\lambda_F^2 m_F}{32\pi^2}\log\left(\frac{\Lambda^2}{m_F^2}\right)+.....$$

Symmetry and the fermion mass

- $\delta m_F \approx m_F$
  - m_F=0, then quantum corrections vanish
  - When m_F=0, Lagrangian is invariant under
    - $\Psi_L \rightarrow e^{i\theta_L} \Psi_L$
    - $\Psi_R \rightarrow e^{i\theta_R} \Psi_R$
  - $m_F \rightarrow 0$ increases the symmetry of the threoy
  - Yukawa coupling (proportional to mass) breaks symmetry and so corrections $\approx m_F$

Scalars are very different

$$-i\Sigma_s(p^2) = -\left(\frac{-i\lambda_F}{\sqrt{2}}\right)^2 (i)^2 \int \frac{d^4k}{(2\pi)^4} \frac{Tr\left[(k+m_F)((k-p)+m_F\right.}{(k^2-m_F^2)[(k-p)^2-m_F^2}$$



$$\delta M_h^2 = \Sigma_S(m_s^2) = -\frac{\lambda_F^2 \Lambda^2}{8\pi^2} + \left(m_s^2 - m_F^2\right)\log\left(\frac{\Lambda}{m_F}\right)$$

$$+(2m_F^2 - \frac{m_s^2}{2})\left(1 + I_1\left(\frac{m_s^2}{m_F^2}\right)\right) + O\left(\frac{1}{\Lambda^2}\right)$$

$$I_1(a) = \int_0^1 dx \ \log\left(1 - ax(1-x)\right)$$

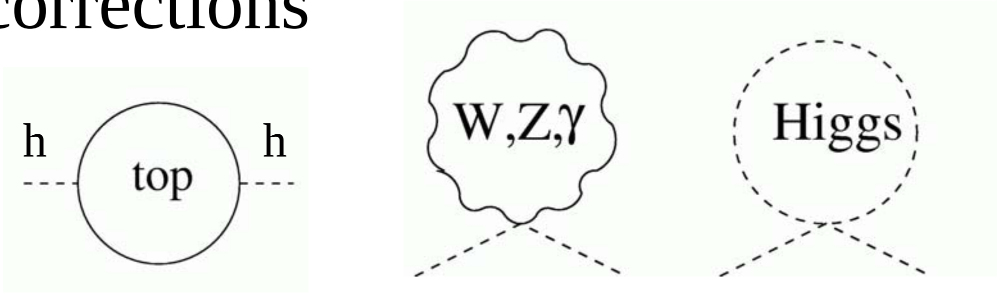
- M_h diverges quadratically!
- This implies quadratic sensitivity to high mass scales

Scalars (#2)

- M_h diverges quadratically!
- Requires large cancellations (hierarchy problem)
- Can do this in Quantum Field Theory
- h does not obey decoupling theorem
  - Says that effects of heavy particles decouple as $M \rightarrow \infty$
- $M_h \rightarrow 0$ doesn't increase symmetry of theory
  - Nothing protects Higgs mass from large corrections

Light Scalars are Unnatural

- Higgs mass grows with scale of new physics, Λ
- No additional symmetry for $M_h=0$, no protection from large corrections



$$\delta M_h^2 = \frac{G_F}{4\sqrt{2}\pi^2}\Lambda^2\left(6M_W^2 + 3M_Z^2 + M_h^2 - 12M_t^2\right)$$

$$= -\left(\frac{\Lambda}{0.7\,\text{TeV}}\,200\,\text{GeV}\right)^2$$

$\boxed{M_h \leq 200 \text{ GeV requires large cancellations}}$

What's the problem?

- Compute M_h in dimensional regularization and absorb infinities into definition of M_h

$$M_h^2 = M_{h0}^2 + \frac{1}{\varepsilon}(\ldots)$$

- Perfectly valid approach
- Except we know there is a high scale

Try to cancel quadratic divergences by adding new particles

- SUSY models add scalars with same quantum numbers as fermions, but different spin
- Little Higgs models cancel quadratic divergences with new particles with same spin

We expect something at the TeV scale

- If it's a SM Higgs then we have to think hard about what the quadratic divergences are telling us
- SM Higgs mass is highly restricted by requirement of theoretical consistency